



## Problem Solving for Irish Second level Mathematicians

Thursday, 19th October 2017

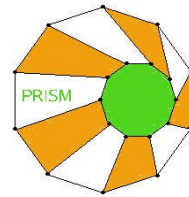
### *PRISM PROBLEMS*

Time allowed: 60 minutes

#### Rules and Guidelines for Contestants

1. You are **not** allowed to use a calculator or any measuring device (e.g. ruler or protractor).
2. **Use a pencil to fill out the answer sheet.** If you make a mistake, you can erase the error and correct it.
3. Write your name clearly (in block capitals) in the space provided in the answer sheet.
4. You should have some extra sheets of your own paper (or a refill pad) for rough work while you are doing the questions.
5. When you have decided on your answer for a particular question, carefully mark your choice for that question on the answer sheet.
6. Do not make any other marks on the answer sheet other than to write your name and to mark your answers to the questions.
7. Some of the questions are quite difficult (especially for junior students who will find the latter part of the paper to be challenging), and we do not expect that many people will have time to think about all of them in 60 minutes. You will probably do better if you concentrate on a few rather than trying to guess the answers to all of the questions. The questions at the beginning are easier than those at the end. The problems are meant to encourage you to think! Don't be in a rush to mark your answer to any of the questions - take your time, read the questions carefully and make sure you understand what is being asked before you start to figure out the answer.
8. **There is no pass/fail mark in PRISM.** Correct answers will score one point each; incorrect or omitted answers will score zero.

*Good luck and thank you for participating in PRISM.  
We hope you will enjoy the problems!*



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*The PRISM Problems for 2017*

- What is the value of  $4 \times 2 - 5$ ?  
(A) 1 (B) 2 (C) 3 (D) 4 (E) -12
- Which of the following is the value of  $\frac{1 \times 5 + 7 - 4 \times 3}{12}$ ?  
(A) 0 (B)  $\frac{11}{12}$  (C) 1 (D)  $\frac{7}{6}$  (E) 2
- What is the lowest common multiple (LCM) of the numbers 3, 8, 9 and 18? (Note: The LCM of these four numbers is the smallest number into which each of them will divide into.)  
(A) 36 (B) 38 (C) 54 (D) 72 (E) 44
- In a class of 26 pupils, the number of boys is 60% more than the number of girls. How many more boys are there than girls in this class of 26 pupils?  
(A) 2 (B) 3 (C) 4 (D) 5 (E) 6
- What is the correct ranking of the numbers  $8^{\frac{4}{3}}$ ,  $9^{\frac{3}{2}}$  and  $27^{\frac{2}{3}}$  in order from smallest to largest?  
(A)  $8^{\frac{4}{3}} < 9^{\frac{3}{2}} < 27^{\frac{2}{3}}$   
(B)  $9^{\frac{3}{2}} < 8^{\frac{4}{3}} < 27^{\frac{2}{3}}$   
(C)  $8^{\frac{4}{3}} < 27^{\frac{2}{3}} < 9^{\frac{3}{2}}$   
(D)  $9^{\frac{3}{2}} < 27^{\frac{2}{3}} < 8^{\frac{4}{3}}$   
(E)  $27^{\frac{2}{3}} < 8^{\frac{4}{3}} < 9^{\frac{3}{2}}$
- Suppose that  $3x = 4y$  and  $16y = z$ . What is  $3x - 8y + \frac{1}{4}z$ ?  
(A) 0 (B) 1 (C) 2 (D) 3 (E) 4
- A rectangle  $ABCD$  has side length  $AB$  given by  $(a + 25)$  cm and side length  $BC$  given by  $a + 5$  cm. If the length of  $AB$  is two times the length of  $BC$ , what is the area in  $\text{cm}^2$  of  $ABCD$ ?  
(A) 160 (B) 450 (C) 600 (D) 800 (E) 1000
- The mean mark of four students in an exam was 55. If two of the students have their marks raised by 5 each, and the other two students have each of their marks lowered by 4, what now will the mean mark of the four students be?

(A) 54 (B) 55 (C) 55.5 (D) 56 (E) 57

9. Let  $a$  be any negative integer. Which of the following numbers has the greatest value?

(A)  $-3a$  (B)  $6a$  (C)  $-a + 1$  (D)  $a$  (E)  $a - 2$

10. In a 24-hour period starting at 9am, how many times do the minute and hour hands coincide (i.e. point in exactly the same direction)?

(A) 23 (B) 22 (C) 21 (D) 20 (E) 12

11. At 12 noon, the minute and hour hands of a clock coincide. What is the first time (in minutes) after midday at which the two hands will again coincide?

(A)  $\frac{720}{11}$  (B)  $\frac{720}{12}$  (C)  $\frac{360}{7}$  (D) 64 (E) 65

12. What is the measure (in degrees) of the angle between two adjacent sides of a regular hexagon (i.e. a shape with 6 equal sides and 6 equal angles)?

(A)  $120^\circ$  (B)  $90^\circ$  (C)  $75^\circ$  (D)  $60^\circ$  (E)  $45^\circ$

13. An operation  $*$  is defined so that  $(a, b) * (c, d) = (ac - bd, ad + bc)$ .  
The value of  $(3, -2) * (1, 4)$  is:

(A)  $(-5, 10)$  (B)  $(-5, 14)$  (C)  $(11, 14)$  (D)  $(11, 10)$  (E)  $(2, 9)$

14. The points  $(p, -3)$  and  $(q, 7)$  are on a line that is perpendicular to the line  $2x + 5y = 9$ .  
What is the value of  $q - p$ ?

(A) -4 (B) 2.5 (C) 4 (D) 10 (E) 25

15. How many three-digit numbers have exactly two of their digits equal?

(A) 666 (B) 667 (C) 450 (D) 343 (E) 243

16. The vertices of the triangle  $ABC$  are the points  $A(1, 2)$ ,  $B(4, 6)$  and  $C(-4, 12)$ . Which one of the following statements about triangle  $ABC$  is true?

- (A)  $ABC$  is a right-angled triangle with the right angle at  $A$
- (B)  $ABC$  is a right-angled triangle with the right angle at  $B$
- (C)  $ABC$  is a right-angle triangle with the right angle at  $C$
- (D)  $ABC$  is not a right triangle but is an equilateral triangle
- (E)  $ABC$  is not a right triangle and is not an equilateral triangle

17. A home heating oil tank has three pumps A, B and C. Pumps A and B are used to pump oil into the tank, and pump C is used to pump oil from the tank into the home (each pump works at its own constant rate). When pump A alone is in operation, an empty tank can be filled by it in 10 minutes. When pump B alone is in operation, an empty tank can be filled in 20 minutes. When pump C alone is in operation, a full tank takes 18,000 minutes to empty. This morning the tank was empty and the three pumps were turned on simultaneously (pumps A and B pumping oil into the tank and pump C pumping oil out). To the nearest minute, how long did it take for the tank to fill up?

(A) 3 (B) 5 (C) 7 (D) 9 (E) 11

18. How many zeros are there at the end of the number  $25! = 25 \times 24 \times 23 \times \dots \times 3 \times 2 \times 1$ ?

(A) 3 (B) 4 (C) 5 (D) 6 (E) 7

19. The equation  $x^3 - 7x + 6 = 0$  has three different solutions. What is the sum of these three numbers?

(A) -4 (B) -2 (C) 0 (D) 2 (E) 4

20. Let  $f(x)$  be a cubic function (i.e.  $f(x) = A + Bx + Cx^2 + Dx^3$ ). Suppose that  $f(1) = 1$ ,  $f(2) = \frac{1}{2}$ ,  $f(3) = \frac{1}{3}$  and  $f(4) = \frac{1}{4}$ . What is  $f(5)$ ?

(A) 0 (B) -1 (C) 1 (D) -2 (E) 2