Problem Solving for Irish Second level Mathematicians

Thursday, 20th October 2016

PRISM PROBLEMS

Time allowed: 60 minutes

Rules and Guidelines for Contestants

1. You are **not** allowed to use a calculator or any measuring device (e.g. ruler or protractor).

2. **Use a pencil to fill out the answer sheet**. If you make a mistake, you can erase the error and correct it.

3. Write your name clearly (in block capitals) in the space provided in the answer sheet.

4. You should have some extra sheets of your own paper (or a refill pad) for rough work while you are doing the questions.

5. When you have decided on your answer for a particular question, carefully mark your choice for that question on the answer sheet.

6. Do not make any other marks on the answer sheet other than to write your name and to mark your answers to the questions.

7. Some of the questions are quite difficult (especially for junior students who will find the latter part of the paper to be challenging), and we do not expect that many people will have time to think about all of them in 60 minutes. You will probably do better if you concentrate on a few rather than trying to guess the answers to all of the questions.
   The questions at the beginning are easier than those at the end.
   The problems are meant to encourage you to think! Don't be in a rush to mark your answer to any of the questions - take your time, read the questions carefully and make sure you understand what is being asked before you start to figure out the answer.

8. **There is no pass/fail mark in PRISM.** Correct answers will score one point each; incorrect or omitted answers will score zero.

Good luck and thank you for participating in PRISM.
We hope you will enjoy the problems!

The PRISM Problems for 2016

1. What is the value of $3 \times 2 + 5$?

    (A) 11   **(B)** 13   **(C)** 15   **(D)** 17   **(E)** 21

2. Each hour, a wood-harvesting machine cuts the same number of logs and places them on a pile. At 8am, there are 10 logs on the pile. At 11am, the pile has grown to 19 logs. How many logs will the pile contain at 6pm?

    (A) 32   **(B)** 34   **(C)** 40   **(D)** 43   **(E)** 66

3. Your friend Ada looked at a digital clock in Sydney, Australia and saw that it read 10:16am. The time there is 10 hours ahead of local time in Ireland. What time will it be here in Ireland exactly 64 hours after Ada looked at the clock?
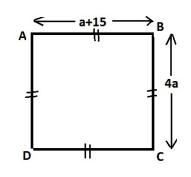
    (A) 4:16am   **(B)** 12:16pm   **(C)** 4:16pm   **(D)** 8:16pm   **(E)** 10:16pm

4. Given that $2^{10} = 1024$, which of the following numbers is closest to one million?

    (A) 2^{20} **(B)** 2^{30} **(C)** 2^{40} **(D)** 2^{100} **(E)** 2^{1010}

5. If $b = 2a$ and $c = 3b$, then $a + b + c$ equals

    (A) $6a$ **(B)** $7a$ **(C)** $8a$ **(D)** $9a$ **(E)** $12a$

6. In the square $ABCD$ shown on the right, the length of the side AB is $(a + 15)$ cm and the length of the side BC is $4a$ cm. How many cm^2 is the area of $ABCD$?

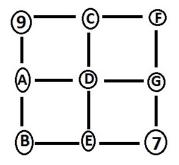
    (A) 400   **(B)** 625   **(C)** 1024   **(D)** 1156   **(E)** 1296

7. Ten years ago, Seosamh was twice as old as his brother. Fourteen years ago he was three times as old as his brother. How old is Seosamh today?
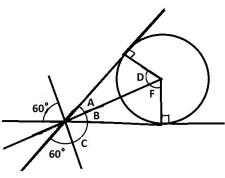
    (A) 36   **(B)** 34   **(C)** 28   **(D)** 26   **(E)** 24

8. Which of the following numbers is the greatest?

    (A) 10^{10000} **(B)** 100^{1000} **(C)** 1000^{100} **(D)** 10000^{10} **(E)** 100000

9. Suppose that on a trip you travel at 100km/h for 70% of the time, 50km/h for 20% of the time, and 10km/h for 10% of the time. What is your overall average speed (in km/h) for the entire trip?

(A) $53\frac{1}{3}$ (B) 65   (C) 72   (D) 81   (E) 90

10. The numbers $1, 2, 3, 4, 5, 6, 7, 8$ and 9 are all to be placed in the nine circles shown on the right so that the sum of the three numbers in each row and column is 15. (The numbers 7 and 9 have already been placed as shown.) Given that $F > G$, what is the value of $(A + E)$?

(A) 7   (B) 8   (C) 10   (D) 11   (E) there is more than one possible value

11. In the diagram on the right, what is the sum of the measures of the angles D and F?

(A) 105°   (B) 120°   (C) 125°   (D) 130°   (E) 135°

12. Which one of these numbers is divisible by 18?

(A) 1234567890   (B) 246802468   (C) 34567345678   (D) 4812162024   (E) 5101520253035

13. You write out the positive integers from 1 to 1,000,000. How many times will the numeral 7 be written?

(A) 200,000   (B) 400,000   (C) 500,000   (D) 600,000   (E) 800,000

14. Ten straight lines are drawn on a two-dimensional plane. Given that three of these lines are parallel to one another, what is the maximum possible number of intersection points formed by the lines?

(A) 27   (B) 42   (C) 49   (D) 52   (E) 70

15. A group of 35 teenagers was surveyed. The information found is summarized here. 24 ride bicycles, 16 use rollerblades, and 10 skateboard while none of these teenagers do all three. All those who skateboard also ride bicycles. Nine of the cyclists neither skateboard nor rollerblade. If all of the teenagers surveyed do at least one of the activities, how many of the teenagers only rollerblade?

(A) 6   (B) 7   (C) 9   (D) 10   (E) 11

16. A box contains a regular coin and a two-headed coin. One of these coins is randomly chosen and then tossed landing heads. What is the probability that the two-headed coin was chosen?

(A) $\frac{1}{2}$ (B) $\frac{5}{8}$ (C) $\frac{2}{3}$ (D) $\frac{3}{4}$ (E) $\frac{7}{8}$

17. In $\triangle\,ABC$, the side AB has length 20 and $\angle ABC = 90°$. If the lengths of the other sides must be positive integers, how many such possible triangles are there?

(A) 7   (B) 6   (C) 5   (D) 4   (E) 3

18. How many ways can the letters of the word LETTERKENNY be arranged in a row if the R must stay in the middle position and the letters L,R,K and Y must remain in their current order LRKY? (An example of an arrangement that meets the requirements is ELTTERENKYN.)

(A) 50400   (B) 25200   (C) 10500   (D) 900   (E) 840

19. 25th April was the earliest day this year for which the product of the month number and the day number is a multiple of 100 (note that $4 \times 25 = 100$). How many *other* days of this year share this property?

(A) 4   (B) 5   (C) 6   (D) 7   (E) 8

20. How many triples (a, b, c) of numbers a, b, c satisfy the equation

$$a^2 + b^2 + c^2 + 1/a^2 + 1/b^2 + 1/c^2 = 6?$$

(A) 2   (B) 3   (C) 4   (D) 6   (E) 8